Analytical Solution of Mixed Convection Flow of a Newtonian Fluid Between Vertical Parallel Plates with Soret, Hall and Ion-Slip...

Article · December 2015

DOI: 10.1007/s40819-015-0127-6

Int. J. Appl. Comput. Math
DOI 10.1007/s40819-015-0127-6

CrossMark

ORIGINAL PAPER

Analytical Solution of Mixed Convection Flow of a Newtonian Fluid Between Vertical Parallel Plates with Soret, Hall and Ion-Slip Effects: Adomian Decomposition Method

Ch. RamReddy[1] · T. Pradeepa[1] · Ch. Venkata Rao[1] · O. Surender[1] · M. Chitra[1]

Abstract The influence of Soret, Hall and Ion-slip effects on the steady, mixed convective heat and mass transfer on fully developed flow in an electrically conducting Newtonian fluid between vertical parallel plates is investigated. The nonlinear governing equations and their associated boundary conditions are initially cast into dimensionless form using similarity transformations and hence solved using Adomian decomposition method. The significance of magnetic parameter, Hall parameter, ion-slip parameter and Soret number on non-dimensional velocities, temperature and concentration profiles are analyzed graphically. Moreover, the numerical data for skin friction, heat and mass transfer rates are shown in tabular form.

Keywords Mixed convection · Newtonian fluid · Hall and Ion Slip effects · Soret effect · ADM

Introduction

The study of mixed convection heat and mass transfer in vertical channel has been the focus of extensive investigation for many decades due to its wide range of applications in the design of cooling systems for electronic devices and in the field of solar energy collection, etc. Further, the heat exchanger technology involves convective flows in vertical channels. Several researchers studied analytically and mostly numerically the problem of mixed convection flow between vertical parallel plates. The problem of flow reversal and heat transfer of fully developed mixed convection in a vertical channel has been discussed by Cheng et al. [8]. Berletta et al. [4] studied mixed convection flow in a fully developed region of a vertical channel with viscous dissipation effect. The analytical and semi-analytical methods are used to solve two-fluid magneto-hydrodynanamic flow and heat transfer in the

✉ Ch. RamReddy
chittetiram@gmail.com; chramreddy@nitw.ac.in

1 Department of Mathematics, National Institute of Technology, Warangal 506004, India

Published online: 22 December 2015

presence of a constant electric field in a vertical channel using Robin boundary conditions by Pratap Kumar et al. [17]. Kaur et al. [14] considered mixed convection flow in vertical parallel plates with reference to laminar, thermal and hydro-dynamical developing flow of Newtonian fluid.

Many researchers discussed the combined free and forced convection flow of an electrically conducting fluid in a channel in the presence of a transverse magnetic field is of special technical significance owing to their many industrial applications such as geothermal reservoirs, cooling of nuclear reactors, thermal insulation, petroleum reservoirs, etc. This type of problem also arises in electronic packages, microelectronic devices during their operations. Alireza and Sahai [1] studied the effect of temperature-dependent transport properties of the developing MHD flow and heat transfer in a parallel-plate channel whose walls were held at constant and equal temperatures. Chamka [6] considered hydromagnetic fully developed laminar mixed convection flow in a vertical channel with symmetric and asymmetric wall heating conditions in the presence or absence of heat generation or absorption effects. The problem of combined free and forced convective magneto-hydrodynamic flow in a vertical channel in the presence of viscous and Ohmic dissipations effects has been analyzed by Umavathi and Malashetty [24]. The laminar mixed convection flow of Newtonian fluid in a plane vertical channel with the effect of MHD has been discussed by Alizadeh and Behanbar [2]. Chand et al. [7] considered an electrically conducting, chemically reacting and radiating viscoelastic fluid through a porous medium in the presence of uniform magnetic field with a constant injection/suction velocity.

In most of the MHD flow problems, the Hall and Ion-slip terms in Ohm's law were ignored. However, in the presence of strong magnetic field, the influence of Hall current and Ion-slip are important. Tani [22] studied the Hall effects on the steady motion of electrically conducting viscous fluid in channels. The steady Couette flow of an electrically conducting viscous incompressible fluid between two parallel horizontal non-conducting porous plates with heat transfer, taking the Ion-slip into consideration has been considered by Attia [3]. A detailed analysis regarding the effect of Hall and Ion-slip effects in non-Newtonian fluids, one can refer the works of Srinivasacharya and Mekonnen [18–20] and Srinivasacharya and Kaladhar [21] (also see the references cited therein). Uddin et al. [23] discussed the influence of thermal radiation on magneto-hydrodynamic heat transfer flow of a micropolar fluid past a non-conducting wedge in the presence of heat source/sink with Hall and Ion-slip effects. Garget al. [10] analyzed the Hall effect for an oscillatory magneto-hydrodynamic convective flow of an incompressible, viscoelastic and electrically conducting fluid through a porous medium filled in a vertical porous channel.

When heat and mass transfer occur simultaneously in a moving fluid, the relations between the fluxes and the driving potentials are of intricate nature. Thermal-diffusion, also called Soret effect, corresponds to species differentiation, developing in an initial homogeneous mixture subjected to a temperature gradient. In most of the studies related to heat and mass transfer process, Soret effect is neglected on the basis that it is a smaller order of magnitude than the effects described by Fourier's and Fick's laws. Eckert and Drak [9] have pointed out that in a convective fluid when the flow of mass is caused by a temperature difference one cannot neglect the thermal-diffusion effect due to its practical application in engineering and science such as hydrology, petrology, geosciences, etc. Kafoussias and Williams [13] presented the diffusion-thermo and thermal diffusion effects on mixed convective and mass transfer on steady laminar boundary layer flow over a vertical flat plate with temperature–dependent viscosity. Rakesh Kumar [15] investigated analytical method for unsteady hydromagnetic mixed convection flow of an incompressible electrically conducting and radiating fluid in a vertical channel filled with a porous medium taking into account the Soret number. Manglesh and Gorla [16] discussed the unsteady free convective flow of an incompressible electrical conducting fluid through a vertical porous channel in the presence of Hall current, radiation, and thermal diffusion effects.

Most of the engineering problems are nonlinear and therefore some of them are solved using numerical methods and some are solved using the different analytic methods. One of the semi-exact methods which does not need linearization or discretization is *Adomian Decomposition Method* (ADM) [see Bellman and Adomian [5]; Adomian (1994)]. This method is a powerful technique, which provides an efficient algorithms for analytic approximate solutions and numeric simulations for real-world applications in the applied sciences and engineering, particularly in the practical solution of linear or nonlinear and deterministic or stochastic operator equations, including ordinary and partial differential equations, integral equations, integro-differential equations, etc. Adomian decomposition method has been employed by Gejji and Jafari [11] to obtain solutions of a system of fractional differential equations and also discussed the convergence of the method. Jafari et al. [12] used the Adomian decomposition method to obtain solutions of fourth-order fractional diffusion-wave equation defined in a bounded space domain by describing the fractional derivative in the Caputo sense.

The aim of the present article is to study mixed convection on fully developed flow of an electrically conducting Newtonian fluid between vertical parallel plates with influence of Soret, Hall and Ion-slip effects. Adomian decomposition method is employed to solve the resulting system of nonlinear equations.

Mathematical Formulation

Consider a fully developed flow of an electrically conducting Newtonian fluid between vertical parallel plates distance $2d$apart. Choose the coordinate system such that x- axis be taken along vertically upward direction through the central line of the channel, y- axis is perpendicular to the plates and the two plates infinitely extended in the direction of x and z. The plates of the channel are at $y = \pm d$. The plate $y = -d$ is maintained at a constant temperature T_1 and concentration C_1, while the plate $y = d$ at a constant temperature T_2 and concentration C_2. The flow is subjected to a uniform magnetic field perpendicular to the flow direction with the Hall and Ion-slip effects. The effect of Hall and Ion-slip current give rise to force in the z-direction, which induces a cross flow in that direction and hence the flow becomes three dimensional. Assume that the flow is steady and the magnetic Reynolds number is very small, so that the induced magnetic field can be neglected in comparison with the applied magnetic field. All the fluid properties are assumed to be constant except for density variations in the buoyancy force term. In addition, the Soret effect is considered. The flow is a mixed convection caused by buoyancy forces and uniform pressure gradient in the direction of x.

With the above assumptions, the governing equations for the fully developed flow of a steady, laminar and incompressible electrically conducting Newtonian fluid are given by

$$\frac{\partial \mathrm{v}}{\partial y} = 0 \Rightarrow v = v_0 \tag{1}$$

$$\rho v \frac{\partial u}{\partial y} = -\frac{\partial p}{\partial x} + \mu \frac{\partial^2 u}{\partial y^2} + \rho g \left[\beta_T (T - T_0) + \beta_C (C - C_0)\right] - \frac{\sigma B_0^2}{\alpha_e^2 + \beta_h^2} (\alpha_e u + \beta_h w) \tag{2}$$

$$\rho v \frac{\partial w}{\partial y} = \mu \frac{\partial^2 w}{\partial y^2} + \frac{\sigma B_0^2}{\alpha_e^2 + \beta_h^2} (\beta_h u - \alpha_e w) \tag{3}$$

$$\rho C_p v \frac{\partial T}{\partial y} = K_f \frac{\partial^2 T}{\partial y^2} + 2\mu \left[\left(\frac{\partial u}{\partial y} \right)^2 + \left(\frac{\partial w}{\partial y} \right)^2 \right] \tag{4}$$

$$v \frac{\partial C}{\partial y} = D \frac{\partial^2 C}{\partial y^2} + \frac{D K_T}{T_m} \frac{\partial^2 T}{\partial y^2} \tag{5}$$

where u, v and w are velocity components along the x, y and z directions respectively, ρ is the density, g is the acceleration due to gravity, C_p is the specific heat, μ is the coefficient of viscosity, σ is the electrical conductivity, B_0 is the magnetic field applied normal to the surface (i.e., in y-direction), K_T is the thermal diffusion ratio, β_h is the Hall parameter, β_i is the ion-slip parameter, $\alpha_e = 1 + \beta_h \beta_i$ is a constant, β_T and β_C are the coefficients of thermal and solutal expansions, K_f is the coefficient of thermal conductivity, α is the thermal diffusivity, D is the mass diffusivity, T_m is the mean fluid temperature and v_0 is a constant.

The boundary conditions are

$$u = 0, w = 0, T = T_1, C = C_1 \text{ at } y = -d \tag{6a}$$

$$u = 0, w = 0, T = T_2, C = C_2 \text{ at } y = d \tag{6b}$$

Introducing the following similarity transformations

$$\mathrm{y} = \eta \mathrm{d}, \mathrm{u} = \mathrm{u_0 U},\ \mathrm{w} = \mathrm{u_0 W},\ \mathrm{T} - \mathrm{T_1} = (\mathrm{T_2} - \mathrm{T_1})\,\theta,\ \mathrm{C} - \mathrm{C_1} = (\mathrm{C_2} - \mathrm{C_1})\,\phi \tag{7}$$

Substituting Eq. (7) in Eqs. (2)–(5), we get the following nonlinear system of ordinary differential equations

$$U'' - RU' - A + \frac{Gr}{\mathrm{Re}}[\theta + B\phi] - \frac{Ha^2}{\alpha_e^2 + \beta_h^2} (\alpha_e U + \beta_h W) = 0 \tag{8}$$

$$W'' - RW' + \frac{Ha^2}{\alpha_e^2 + \beta_h^2} (\beta_h U - \alpha_e W) = 0 \tag{9}$$

$$\theta'' - R\,\mathrm{Pr}\,\theta' + 2Br \left[\left(U'\right)^2 + \left(W'\right)^2 \right] = 0 \tag{10}$$

$$\phi'' - RSc\phi' + S_r Sc\theta'' = 0 \tag{11}$$

where primes denote differentiation with respect to η, $\mathrm{Re} = \frac{\rho u_0 d}{\mu}$ is the Reynolds number, $R = \frac{\rho v_0 d}{\mu}$ is the suction/injection parameter, $\mathrm{Pr} = \frac{\mu C_p}{K_f}$ is the Prandtl number, $Sc = \frac{v}{D}$ is the Schmidt number, $Ha = B_0 d \sqrt{\frac{\sigma}{\mu}}$ is the Hartmann number, $Gr = \frac{\rho^2 g \beta_T d^3}{\mu^2}(T_2 - T_1)$ is the Grashof number,$Br = \frac{\mu u_0^2}{K_f (T_2 - T_1)}$ is the Brinkman number, $A = \frac{d^2}{\mu u_0} \frac{dp}{dx}$ is the constant pressure gradient, $B = \frac{\beta_C (C_2 - C_1)}{\beta_T (T_2 - T_1)}$ is the Buoyancy ratio and $S_r = \frac{D K_T (T_2 - T_1)}{T_m \upsilon (C_2 - C_1)}$ is the Soret number.

Boundary conditions in terms of U, W, θ, ϕ become

$$\mathrm{U} = 0, \quad \mathrm{W} = 0, \quad \theta = \mathrm{r_T}, \quad \phi = \mathrm{r_C} \quad \text{at} \quad \eta = -1 \tag{12a}$$

$$\mathrm{U} = 0, \quad \mathrm{W} = 0, \quad \theta = 1, \quad \phi = 1 \quad \text{at} \quad \eta = 1 \tag{12b}$$

Skin Friction, Heat and Mass Transfer Coefficients

The shear stress, heat and mass transfer coefficients are given by

$$\tau = \mu \left(\frac{\partial u}{\partial y} \right)_{y=\pm d}, \quad q_w = -k \left(\frac{\partial T}{\partial y} \right)_{y=\pm d} \quad \text{and} \quad q_m = -D \left(\frac{\partial C}{\partial y} \right)_{y=\pm d}$$

The non-dimensional skin friction coefficients C_{f_1} and C_{f_2}, heat and mass transfer rates Nu_1, Nu_2, Sh_1andSh_2 respectively on the lower and upper plates are given by

$$C_{f_1} = 2U'(-1) \quad \text{and} \quad C_{f_2} = 2U'(1) \tag{13a}$$

$$Nu_1 = \theta'(-1), \quad Nu_2 = \theta'(1) \quad \text{and} \quad Sh_1 = \phi'(-1), \; Sh_2 = \phi'(1) \tag{13b}$$

Analytical Solution Via ADM

Consider the equation $Fu(t) = g(t)$, where F represents a general nonlinear ordinary or partial differential operator including both linear and nonlinear terms. The linear terms are decomposed into $L + R$, where L is easily invertible (usually the highest order derivative) and R is the remained of the linear operator.

Thus, the equation can be written as

$$Lu + Nu + Ru = g \tag{14}$$

where *Nu* indicates the nonlinear terms.

By solving this equation for *Lu*, since L is invertible, we can write

$$L^{-1}Lu = L^{-1}g - L^{-1}Ru - L^{-1}Nu \tag{15}$$

If L is a second-order operator, L^{-1} is a twofold indefinite integral. By solving Eq. (15), we have

$$u = A + Bt + L^{-1}g - L^{-1}Ru - L^{-1}Nu \tag{16}$$

where A and B are constants of integration and can be found from the boundary or initial conditions. Adomian decomposition method assumes the solution u that can be expanded into infinite series as

$$u = \sum_{n=0}^{\infty} u_n \tag{17}$$

Also, the nonlinear term *Nu* will be written as

$$Nu = \sum_{n=0}^{\infty} A_n \tag{18}$$

where A_n are the special Adomian polynomials. By specified A_n, next component of u can be determined

$$u_{n+1} = L^{-1} \sum_{n=0}^{\infty} A_n \tag{19}$$

Finally, after some iteration and getting sufficient accuracy, the solution can be expressed by Eq. (16). In Eq. (19), the Adomian polynomials can be generated by several means. Here we used the following recursive formulation

$$A_n = \frac{1}{n!}\left[\frac{d^n}{d\lambda^n}\left[N\left(\sum_{i=0}^{n}\lambda^i u_i\right)\right]\right]_{\lambda=0}, n = 0, 1, 2, \ldots. \tag{20}$$

Since the method does not resort to linearization or assumption of weak nonlinearity, the solution generated in general more realistic than those achieved by simplifying the model of the physical problem.

According to Eq. (15), Eqs. (8)–(11) must be written as following

$$L_1 U = RU' + A - \frac{Gr}{\mathrm{Re}}[\theta + B\phi] + \frac{Ha^2}{\alpha_e^2 + \beta_h^2}(\alpha_e U + \beta_h W) \tag{21}$$

$$L_2 W = RW' - \frac{Ha^2}{\alpha_e^2 + \beta_h^2}(\beta_h U - \alpha_e W) \tag{22}$$

$$L_3 \theta = R\,\mathrm{Pr}\,\theta' - 2Br\left[\left(U'\right)^2 + \left(W'\right)^2\right] \tag{23}$$

$$L_4 \phi = RSc\phi' - S_r Sc\theta'' \tag{24}$$

where the differential operator L_1, L_2, L_3 and L_4 are given by $L_1 = L_2 = L_3 = L_4 = \frac{d^2}{d\eta^2}$. Assume the inverse of the operator L_1^{-1}, L_2^{-1}, L_3^{-1}andL_4^{-1} exists and it can be integrated from 0 to η, i.e. $L_1^{-1} = L_2^{-1} = L_3^{-1} = L_4^{-1} = \int_0^{\eta}\int_0^{\eta}(.)d\eta d\eta$.

After operating L_1^{-1}, L_2^{-1}, L_3^{-1}, L_4^{-1}on Eqs. (21) to (24) and exerting boundary condition on it, we have

$$U(\eta) = U(0) + U'(0)\eta + L_1^{-1}N_1 u \tag{25}$$

$$W(\eta) = W(0) + W'(0)\eta + L_2^{-1}N_2 u \tag{26}$$

$$\theta(\eta) = \theta(0) + \theta'(0)\eta + L_3^{-1}N_3 u \tag{27}$$

$$\phi(\eta) = \phi(0) + \phi'(0)\eta + L_4^{-1}N_4 u \tag{28}$$

where

$$N_1 u = RU' + A - \frac{Gr}{\mathrm{Re}}[\theta + B\phi] + \frac{Ha^2}{\alpha_e^2 + \beta_h^2}(\alpha_e U + \beta_h W); N_2 u = RW' - \frac{Ha^2}{\alpha_e^2 + \beta_h^2}(\beta_h U - \alpha_e W);$$

$$N_3 u = R\,\mathrm{Pr}\,\theta' - 2Br\left[\left(U'\right)^2 + \left(W'\right)^2\right]; N_4 u = RSc\phi' - S_r Sc\theta''.$$

The ADM introduced the following expression

$$U(\eta) = \sum_{m=0}^{\infty} U_m(\eta) = U_0(\eta) + L_1^{-1}N_1 u \tag{29}$$

$$W(\eta) = \sum_{m=0}^{\infty} W_m(\eta) = W_0(\eta) + L_2^{-1}N_2 u \tag{30}$$

$$\theta(\eta) = \sum_{m=0}^{\infty} \theta_m(\eta) = \theta_0(\eta) + L_3^{-1} N_3 u \tag{31}$$

$$\phi(\eta) = \sum_{m=0}^{\infty} \phi_m(\eta) = \phi_0(\eta) + L_4^{-1} N_4 u \tag{32}$$

To determine the components of $U_m(\eta)$, $W_m(\eta)$, $\theta_m(\eta)$ and $\phi_m(\eta)$, the initial values of $U_0(\eta)$, $W_0(\eta)$, $\theta_0(\eta)$ and $\phi_0(\eta)$ are defined by applying the boundary conditions

$$U_0(\eta) = a_1 + a_2\eta, \quad W_0(\eta) = a_3 + a_4\eta, \quad \theta_0(\eta) = a_5 + a_6\eta, \quad \phi_0(\eta) = a_7 + a_8\eta \tag{33}$$

$$U_1(\eta) = \left[Ra_2 + A - \frac{Gr}{\mathrm{Re}}(a_5 + Ba_7) + \frac{Ha^2}{\alpha_e^2 + \beta_h^2}(\alpha_e a_1 + \beta_h a_3) \right] \frac{\eta^2}{2} + \left[-\frac{Gr}{\mathrm{Re}}(a_6 + Ba_8) + \frac{Ha^2}{\alpha_e^2 + \beta_h^2}(\alpha_e a_2 + \beta_h a_4) \right] \frac{\eta^3}{6} \tag{34}$$

$$W_1(\eta) = \left[Ra_4 - \frac{Ha^2}{\alpha_e^2 + \beta_h^2}(\beta_h a_1 - \alpha_e a_3) \right] \frac{\eta^2}{2} - \frac{Ha^2}{\alpha_e^2 + \beta_h^2}(\beta_h a_2 - \alpha_e a_4)\frac{\eta^3}{6} \tag{35}$$

$$\theta_1(\eta) = \left[R\,\mathrm{Pr}\, a_6 - 2Br(a_2^2 + a_4^2) \right] \frac{\eta^2}{2} \tag{36}$$

$$\phi_1(\eta) = [R\ Sca_8]\frac{\eta^2}{2} \tag{37}$$

$$\phi_2(\eta) = R^2 Sc^2 a_8 \frac{\eta^3}{6} - S_r Sc \left[R\,\mathrm{Pr}\, a_6 - 2Br(a_2^2 + a_4^2) \right] \frac{\eta^2}{2} \tag{38}$$

and $U_m(\eta)$,$W_m(\eta)$, $\theta_m(\eta)$and $\phi_m(\eta)$ for $m \geq 2$ be determined in similar way.

Then using the above in the following series expansions

$$U(\eta) = \sum_{m=0}^{\infty} U_m(\eta), \quad W(\eta) = \sum_{m=0}^{\infty} W_m(\eta), \quad \theta(\eta) = \sum_{m=0}^{\infty} \theta_m(\eta), \quad \phi(\eta) = \sum_{m=0}^{\infty} \phi_m(\eta) \tag{39}$$

lead to following equations

$$U(\eta) = a_1 + a_2\eta + \left[Ra_2 + A - \frac{Gr}{\mathrm{Re}}(a_5 + Ba_7) + \frac{Ha^2}{\alpha_e^2 + \beta_h^2}(\alpha_e a_1 + \beta_h a_3) \right] \frac{\eta^2}{2} + \left[-\frac{Gr}{\mathrm{Re}}(a_6 + Ba_8) + \frac{Ha^2}{\alpha_e^2 + \beta_h^2}(\alpha_e a_2 + \beta_h a_4) \right] \frac{\eta^3}{6} + \cdots \tag{40}$$

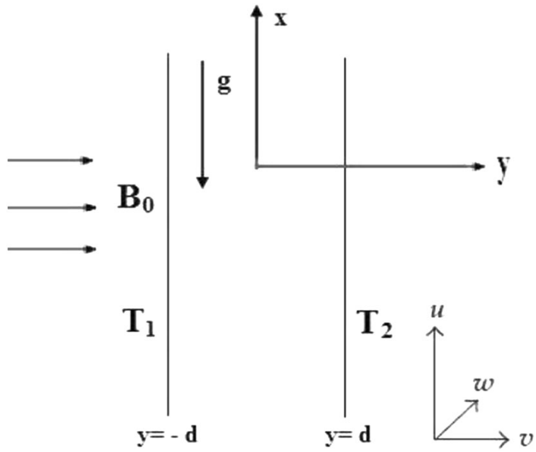
$$W(\eta) = a_3 + a_4\eta + \left[Ra_4 - \frac{Ha^2}{\alpha_e^2 + \beta_h^2}(\beta_h a_1 - \alpha_e a_3) \right] \frac{\eta^2}{2} - \frac{Ha^2}{\alpha_e^2 + \beta_h^2}(\beta_h a_2 - \alpha_e a_4)\frac{\eta^3}{6} + \cdots \tag{41}$$

$$\theta(\eta) = a_5 + a_6\eta + \left[R\,\mathrm{Pr}\, a_6 - 2Br(a_2^2 + a_4^2) \right] \frac{\eta^2}{2} + \cdots \tag{42}$$

$$\phi(\eta) = a_7 + a_8\eta + \left[RSca_8 - S_r Sc[R\,\mathrm{Pr}\, a_6 - 2Br(a_2^2 + a_4^2)] \right] \frac{\eta^2}{2} + R^2 Sc^2 a_8 \frac{\eta^3}{6} + \cdots \tag{43}$$

The accuracy of ADM solution increases by increasing the number of solution terms (m). For the complete solution of equations above $a_1, a_2, a_3, a_4, a_5, a_6, a_7$ and a_8 should be determined, with boundary conditions.

Fig. 1 Physical model and coordinate system



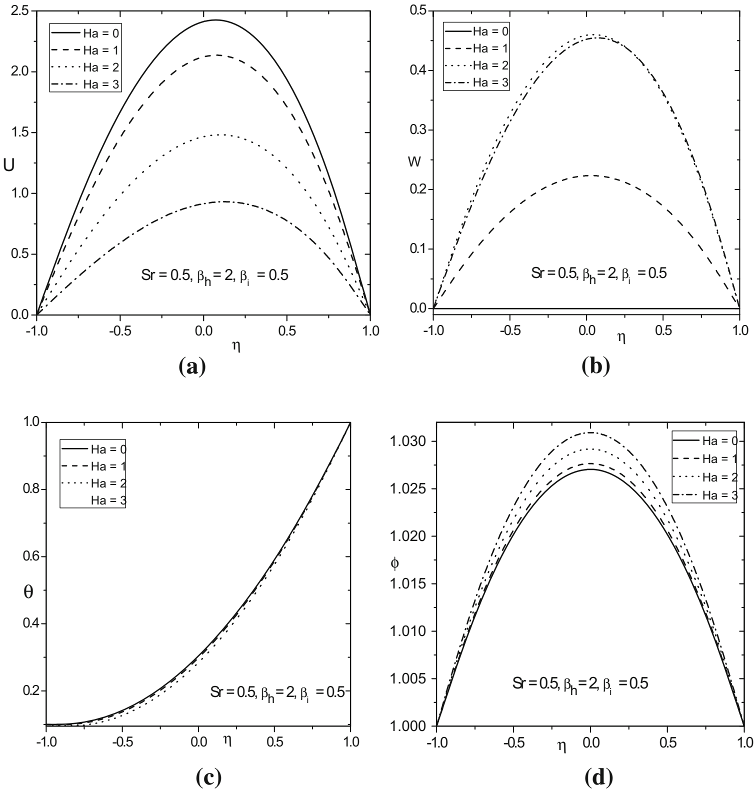


Fig. 2 Effect of Hartmann parameter on **a** velocity, **b** induced velocity, **c** temperature, **d** concentration profiles

Results and Discussion

The solutions for $U(\eta)$, $W(\eta)$, $\theta(\eta)$ and $\phi\ (\eta)$ have been computed and shown graphically in Figs. 1, 2, 3 and 4. In order to study the effects of magnetic parameter (*Ha*), Hall parameter (β_h), Ion-slip parameter (β_i) and Soret parameter (*Sr*) computations are carried out by taking *Pr = 0.72, Sc=0.22,* $r_T = 0.1$, $r_C = 1.0$, $Gr = 10$, $Re = 2$, $R = 2$, $Br = 0.5$, $A = 1$ and $B = 1$.

Figure 2a reveals the effect of the magnetic parameter *Ha* on $U(\eta)$. It can be observed that the velocity $U(\eta)$ decreases with an increase in magnetic parameter *Ha*. This is due to the fact that, the introduction of a transverse magnetic field, normal to the flow direction, has a tendency to create the drag known as the Lorentz force which tends to resist the flow. Hence the velocity decreases as the magnetic parameter *Ha* increases. The effect of *Ha* on the induced flow in *z*-direction $W(\eta)$ is shown in Fig. 2b. It can be seen from this figure that $W(\eta)$ increases with an increase in parameter *Ha*. Figure 2c depicts the variation of temperature

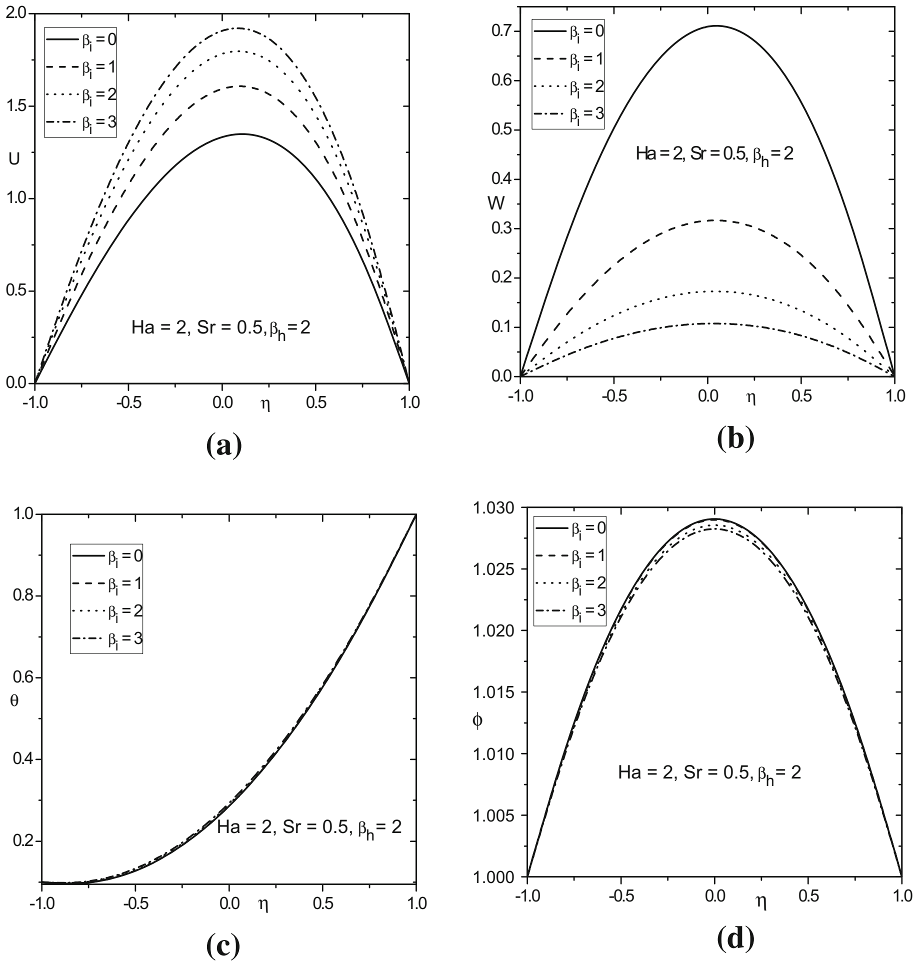


Fig. 3 Effect of ion-slip parameter on **a** velocity, **b** induced velocity, **c** temperature, **d** concentration profiles

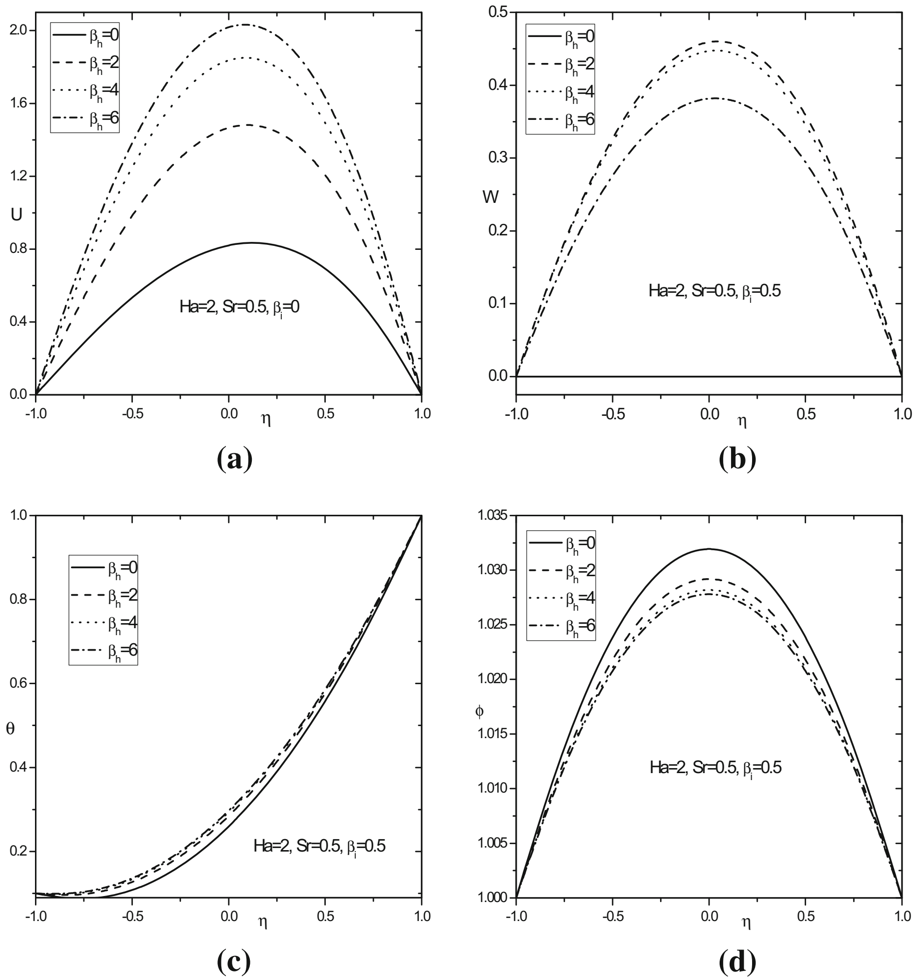


Fig. 4 Effect of Hall parameter on **a** velocity, **b** induced velocity, **c** temperature, **d** concentration profiles)

with Ha. The temperature $\theta(\eta)$ decreases with an increase in parameter Ha. As explained above, the transverse magnetic field gives rise to a resistive force known as the Lorentz force of an electrically conducting fluid. This force makes the fluid experience a resistance by increasing the friction between its layers and thus decreases its temperature. Figure 2d illustrates the variation of concentration with the effect of magnetic parameter Ha. As the magnetic parameter Ha increases, the concentration $\phi\ (\eta)$ increases due to the existence of Lorentz force of an electrically conducting fluid.

Figures 3a–d represent the effect of the Ion-slip parameter (β_i) on velocity components $U(\eta)$ and $W(\eta)$, temperature $\theta(\eta)$ and concentration $\phi\ (\eta)$. It can be seen from these figures that the velocity $U(\eta)$ increases with an increase of the ion-slip parameter β_i. The induced flow in the z-direction decreases with an increase in the parameter β_i. The temperature $\theta(\eta)$ increases with an increase in parameter β_i. As Ion-slip parameter increases the effective conductivity also increases, in turn, decreases the damping force on the velocity component in the direction of the flow, and hence the velocity component in the flow direction increases. The concentration $\phi\ (\eta)$decreases with rise of the Ion-slip parameter.

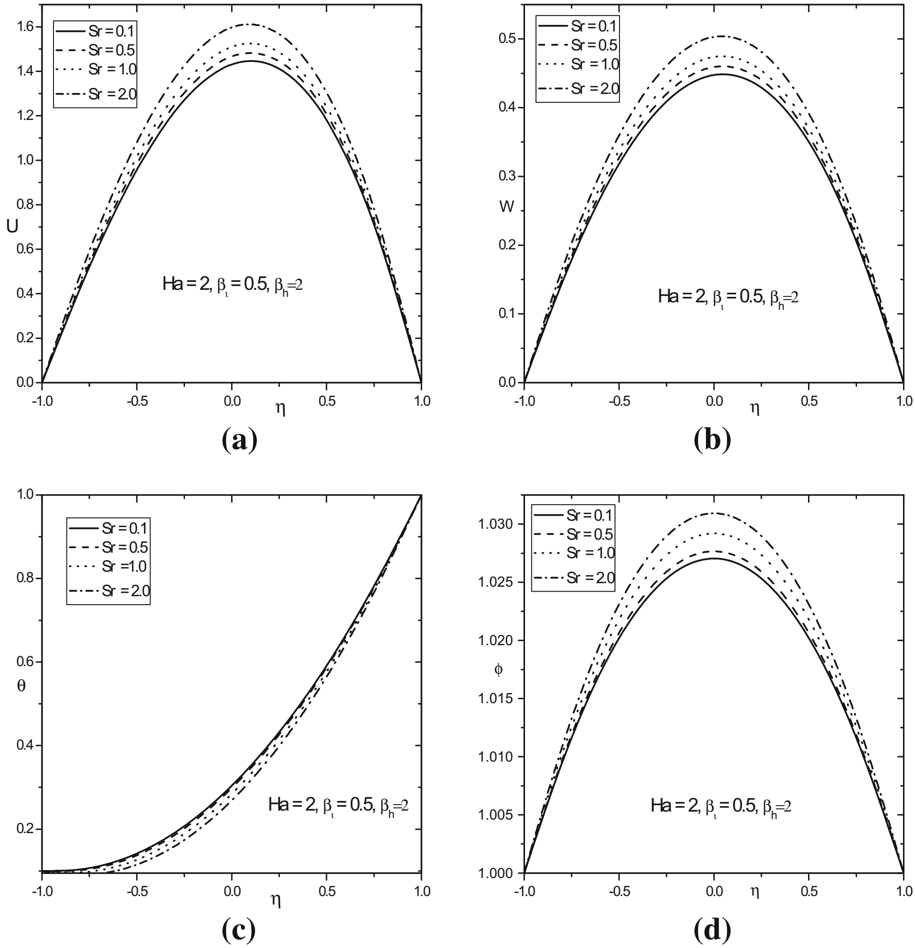


Fig. 5 Effect of Soret number on **a** velocity, **b** induced velocity, **c** temperature, **d** concentration profiles

The variation of velocity components $U(\eta)$ and $W(\eta)$, temperature $\theta(\eta)$ and concentration $\phi(\eta)$ with Hall parameter (β_h)are shown in Figs. 4a–d. The dimensionless velocity component $U(\eta)$ and temperature $\theta(\eta)$ increase with an increase in the Hall parameter (β_h) as depicted in Fig. 4a, c. Initially the induced velocity component is flat and then decreases with increase of Hall parameter. The concentration decreases within crease of Hall parameter (β_h) as shown in Fig. 4d.The inclusion of Hall parameter decreases the resistive force imposed by the magnetic field due to its effect in reducing the effective conductivity. Hence, the velocity component $U(\eta)$ and temperature $\theta(\eta)$ increase but, the induced velocity $W(\eta)$ in the absence of initial point ($\beta_h = 0$) and concentration $\phi(\eta)$ decreases with an increase of the Hall parameter.

Figures 5a–d display the Soret effect on non dimensional velocity components $U(\eta)$ and $W(\eta)$, temperature $\theta(\eta)$ and concentration $\phi(\eta)$. It can be seen from Fig. 5a, b that velocity components $U(\eta)$ and $W(\eta)$ increase with rise of Soret parameter. The temperature decreases with increase of Soret parameter as depicted in Fig. 5c. Figure 5d shows that with increase of Soret parameter the concentration increases.

Table 1 Effects of skin friction, heat and mass transfer coefficients for varying values of MHD, Soret, hall and ion-slip effects

Ha	β_i	β_h	Sr	C_{f_1}	C_{f_2}	Nu_1	Nu_2	Sh_1	Sh_2
0	0.5	2	0.5	4.14112	−5.64112	0.048375	−0.948375	−0.0548213	0.0548213
1	0.5	2	0.5	3.58400	−5.02170	0.059607	−0.959607	−0.0560568	0.0560568
2	0.5	2	0.5	2.34210	−3.60210	0.087750	−0.987750	−0.0591525	0.0591525
3	0.5	2	0.5	1.34774	−2.36313	0.119769	−1.019770	−0.0626746	0.0626746
2	0.0	2	0.5	2.07404	−3.32814	0.0852602	−0.985260	−0.0588786	0.0588786
2	1.0	2	0.5	2.58415	−3.87396	0.0842030	−0.984203	−0.0587623	0.0587623
2	2.0	2	0.5	2.94207	−4.28515	0.0760673	−0.976067	−0.0578674	0.0578674
2	3.0	2	0.5	3.17688	−4.55476	0.0702777	−0.970278	−0.0572305	0.0572305
2	0.5	0	0.5	1.21362	−2.11362	0.1383750	−1.038380	−0.0647213	0.0647213
2	0.5	2	0.5	2.34210	−3.60210	0.0877500	−0.987750	−0.0591525	0.0591525
2	0.5	4	0.5	3.03505	−4.41052	0.0696014	−0.969601	−0.0571562	0.0571562
2	0.5	6	0.5	3.38255	−4.80178	0.0624375	−0.962438	−0.0563681	0.0563681
2	0.5	2	0.1	2.27112	−3.53112	0.08775	−0.98775	−0.0118305	0.0118305
2	0.5	2	0.5	2.34210	−3.60210	0.08775	−0.98775	−0.0591525	0.0591525
2	0.5	2	1.0	2.43083	−3.69083	0.08775	−0.98775	−0.1183050	0.1183050
2	0.5	2	2.0	2.60829	−3.86829	0.08775	−0.98775	−0.2366100	0.2366100

The variation of local skin friction coefficient, heat and mass transfer rates with effects of magnetic parameter (Ha), Hall parameter (β_h), Ion-slip parameter (β_i) and Soret parameter are shown in Table 1 for both lower and upper plates. For fixed values of $\beta_h = 2$, $\beta_i = 0.5$ and $Sr = 0.5$, the skin friction coefficient and mass transfer rate decrease whereas heat transfer increases with increase of magnetic parameter at lower plate but at upper plate the skin friction, heat and mass transfer rates show opposite trend. From Table 1 it can observe that with increase of Ion-slip parameter for fixed parameter $Ha = 2$, $\beta_h = 2$ and $Sr = 0.5$ the skin friction coefficient and mass transfer rate increase and heat transfer rate decreases at lower plate whereas for upper plate the skin friction coefficient, heat and mass transfer rates show reverse trend. As resulted in Table 1 that for fixing the other parameters $Ha = 2$, $\beta_i = 0.5$ and $Sr = 0.5$ the skin friction and mass transfer rate are increasing and heat transfer rate is decreasing with increasing of Hall parameter at lower plate but, it shows opposite trend for upper plate. Table 1 reads that the skin friction coefficient and mass transfer rate are showing opposite trend but heat transfer rate did not show any significant effect at the lower plate whereas at upper plate skin friction coefficient decreases, mass transfer rate increases and heat transfer rate did not show any effect with increase of Soret parameter for fixed parameters $\beta_h = 2$, $\beta_i = 0.5$ and $Ha = 2$.

Conclusions

In this paper, the Soret, Hall and Ion-slip effects on fully developed flow of an electrically conducting Newtonian fluid between vertical parallel plates has been studied. The governing equations are expressed in the non-dimensional form and are solved using Adomian decomposition method. The features of flow characteristics are analysed by plotting graphs and discussed in detail. The main findings are summarized as follows:

- As the magnetic parameter increases the induced velocity, concentration, skin friction and mass transfer rate of the upper plate and heat transfer rate at lower plate are higher, while it is lower for velocity in the direction of flow, temperature, skin friction, mass transfer rate at lower plate and heat transfer rate at upper plate.
- It is observed that the velocity in the flow direction, temperature, skin friction and mass transfer rate at lower plate and heat transfer rate at upper plate increase, but the induced velocity, concentration, heat transfer rate at lower plate, skin friction and mass transfer rate at upper plate decrease with an increase of Ion-slip parameter.
- The velocity in the flow direction, temperature, skin fiction and mass transfer rate at lower plate and heat transfer rate at upper plate are increasing, the induced velocity in the absence of a Hall parameter at zero, concentration, heat transfer rate at lower plate, skin friction and mass transfer rate at upper plate are decreasing with increasing of Hall parameter.
- It is concluded that in the presence of Soret parameter, velocity in the flow direction, induced velocity, concentration are more whereas the temperature is less. We noticed that skin friction and mass transfer rate are showing opposite trend, but heat transfer rate did not show any significant effect at both lower and upper plates.

References

1. Alireza, S., Sahai, V.: Heat transfer in developing magneto-hydrodynamic Poiseuille flow and variable transport properties. Int. J. Heat Mass Transf. **33**, 1711–1720 (1990)
2. Alizadeh, R., Behanbar, A.D.: The effect of MHD on laminar mixed convection of Newtonian fluid between vertical parallel plates channel. WSEAS Trans. Heat Mass Transf. **8**, 139–145 (2013)
3. Attia, H.A.: Unsteady couette flow with heat transfer considering Ion slip. Turk J. Phys. **29**, 379–388 (2005)
4. Barletta, A., Magyari, E., Keller, B.: Dual mixed convection flows in a vertical channel. Int. J. Heat Mass Transf. **48**, 4835–4845 (2005)
5. Bellman, R.E., Adomian, G.: Partial Differential Equations: New Methods for their Treatment and Solution. D. Reidel, Dordrecht (1985)
6. Chamkha, A.J.: On laminar hydromagnetic mixed convection flow in a vertical channel with symmetric and asymmetric wall heating conditions. Int. J. Heat Mass Transf. **45**, 2509–2525 (2002)
7. Chand, K., Kumar, D., Kumar, S.: MHD flow of radiating and chemically reacting viscoelastic fluid through a porous medium in porous vertical channel with constant suction. Int. J. Eng. Sci. Invent. **2**, 1–7 (2013)
8. Cheng, C.H., Kou, H.S., Huang, W.H.: Flow reversal and heat transfer of fully developed mixed convection in vertical channels. J. Thermophys. Heat Transf. **3**(375–83), 29 (1990)
9. Eckert, E.R.G., Drake, R.M.: Analysis of Heat and Mass Transfer. McGraw-Hill, New York (1972)
10. Garg, B.P., Singh, K.D., Bansal, A.K.: Hall current effect on viscoelastic (Walter's liquid model-B) MHD oscillatory convective channel flow through a porous medium with heat radiation. Kragujevac J. Sci. **36**, 19–32 (2014)
11. Gejji, V.D., Jafari, H.: Adomian decomposition: a tool for solving a system of fractional differential equations. J. Math. Anal. Appl. **301**(2), 508–518 (2005)
12. Jafari, H., Dehghan, M., Sayevand, K.: Solving a fourth-order fractional diffusion-wave equation in a bounded domain by decomposition method. Numer. Methods Partial Differ. Equ. **24**(4), 1115–1126 (2008)
13. Kafoussias, N.G., Williams, N.G.: Thermal-diffusion and diffusion-thermo effects on mixed free-forced convective and mass transfer boundary layer flow with temperature dependent viscosity. Int. J. Eng. Sci. **33**, 1369–1384 (1995)
14. Kaur, P.P., Agrawal, S.P., Kumar, A.: Analytical study of MHD on laminar mixed convection of Newtonian fluid between vertical parallel plates through channel. Int. J. Multidiscip. Res. Adv. Eng. **6**(1), 85–98 (2014)

15. Kumar, R.: Thermo diffusion and radiative heat effects on unsteady oscillatory mixed convection MHD flow through a planar channel filled with porous medium. Int. J. Phys. Math. Sci. **3**(1), 1–13 (2012)
16. Mangleh, A., Gorla, M.G.: MHD free convective flow through porous medium in the presence of hall current, radiation and thermal diffusion. Indian J. Pure Appl. Math. **44**(6), 743–756 (2013)
17. Prathap Kumar, J., Umavathi, J.C., Chamkha, A.J., Ramarao, Y.: Mixed convection of electrically conducting and viscous fluid in a vertical channel using robin boundary conditions. Can. J. Phys. (2014). doi:10.1139/cjp-2014-0072
18. Srinivasacharya, D., Shiferaw, M.: Hall and ion-slip effects on the flow of micropolar fluid between parallel plates. Int. J. Appl. Mech. Eng. **13**(1), 251–262 (2008a)
19. Srinivasacharya, D., Shiferaw, M.: MHD flow of a micropolar fluid in a rectangular duct with Hall and Ion slip effects. J. Braz. Soc. Mech. Sci. Eng. **30**(4), 313–318 (2008)
20. Srinivasacharya, D., Shiferaw, M.: MHD flow of a micropolar fluid in a circular pipe with hall effects. ANZIAM J. **51**, 277–285 (2009)
21. Srinivasacharya, D., Kaladhar, K.: Mixed convection flow of couple stress fluid between parallel vertical plates with Hall and Ion-slip effects. Commun. Nonlinear Sci. Numer. Simul. **17**, 2447–2462 (2012)
22. Tani, I.: Steady flow of conducting fluids in channels under transverse magnetic fields with consideration of Hall effects. J. Aerosp. Sci. **29**, 297–305 (1962)
23. Uddin, Z., Kumar, M., Harmand, S.: Influence of thermal radiation and heat generation/absorption on MHD heat transfer flow of a micropolar fluid past a wedge with hall and ion-slip currents. Therm. Sci. **18**(2), S489–S502 (2014)
24. Umavathi, J.C., Malashetty, M.S.: Magneto-hydrodynamic mixed convection in a vertical channel. Int. J. Non-Linear Mech. **40**, 91–101 (2005)